PERSONALITY TRAITS AND THE PREFERENCE TO DO NONROUTINE
MATHEMATICS PROBLEMS

BY

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To my parents and my wife

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The purpose of this study was to investigate possible relation-
ships between students' personality preferences and the type of mathe-
matical problems they preferred to do. The Myers-Briggs Type Indicator
(MBTI) was used to assess personality preferences, and a set of 24
nonroutine mathematics problems was used to assess perceived discrimi-
nations among the problem types. The problems varied in terms of both
content and structure. The four categories of problems were number
theoretic problems that could be solved using an inductive strategy,
number theoretic problems that could be solved using factors or other
properties of the quantities involved, logic problems, and problems in
goemetry.

Eighteen above-average high school students, who were enrolled
in a special course designed to teach problem solving processes, partici-
pated in the study. Throughout the course, the heuristic processes
necessary to solve the types of problems used in this study were

stressed. At the conclusion of the lessons, the students were given the set of 24 problems and asked to sort them from the one they would most like to do to the one they would least like to do.

For purposes of analysis, the students were grouped by personality type. Intragroup similarities were investigated using the Friedman test, while intergroup differences were investigated using Spearman's rho. The results indicated that the group, as a whole, tended to sort the problems in the same manner. Personality style, as measured by the MBTI, was not a discriminating factor in determining the outcome of the ordering process.

The results did support the conjecture that the students would sort the problem set according to the content or structure of the problem. Problems that contained either a logic content or structure were preferred over all other types as demonstrated by their being picked as choices 1 through 5. Inductive problems not connected to the geometry-content problems were the 2nd most preferred while factor problems not connected with the logic-content problems were 3rd. The least preferred problems were those with a geometry content and an inductive structure. These problem types had a mean rank which placed them in 5 of the last 6 positions.

Problem solving has become one of the most important issues in modern curriculum development. This interest has helped to focus increased attention on one of the most important areas of mathematics education. Part of this new emphasis is the reaction of the mathematics education community to the increased attention to basic skills programs brought about by decreased student performance throughout the country. Many educators have been apprehensive that mathematics classrooms would become intellectual wastelands, where only simple computations were taught. As a result of this reaction many groups came to the front espousing problem solving as a most important basic skill. Among these groups are the National Institute of Education's Conference on Basic Mathematical Skills and Learning which produced the EUCLID report (NIE, 1975) and the National Advisory Committee on Mathematical Education which produced the NACOME report (NACOME, 1975). The working group on goals for basic mathematical skills and learning, in the EUCLID report, stated, "Problem solving should be considered a special goal interrelated with all of the . . . goals presented here" (NIE, 1975, Vol. II, p. 20).

Background

The National Council of Teachers of Mathematics (NCTM) in its monograph entitled, _An Agenda for Action: Recommendations for_

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School Mathematics of the 1980s, listed as its first recommendation that "problem solving be the focus of school mathematics in the 1980s" (1980, p. 1). The priorities in School Mathematics Project (PRISM), which was funded by the National Science Foundation through the NCTM, was designed to collect information on current beliefs and reaction to possible changes in mathematics instruction. Among other results, the PRISM researchers found that

1. The data clearly indicate widespread support for problem solving in the school mathematics program.

2. Problem solving was consistently ranked high in priority for increased emphasis in the 1980s.

3. If a limited amount of money could be spent for the development of new materials, all samples would choose problem solving as having top priority for the expenditure of the funds.

This reemphasis on problem solving in instruction has also been evidenced by an increase in the writing on the subject in special topic journals such as the November, 1977, issue of the Arithmetic Teacher, the March, 1978, issue of School Science and Mathematics, as well as articles in nondiscipline journals, such as the April, 1980, issue of Educational Leadership.

Research

The modern evolution of the study of problem solving as a process began with the work of Polya (1957). Using Polya's concept of a modern heuristic, many investigators attempted to delineate those

factors which affect student problem-solving performance. The works
of Wilson (1967) and Kilpatrick (1967) have opened important avenues
to the understanding of the problem-solving process. Growth in this
area has accelerated and the quality of research has improved. The
works of Kantowski (1974), Goldin (1979), Webb (1977), and many others
have brought about a coherent and reasonable approximation of those
factors which affect the problem-solving process.

Statement of the Problem

There is a trend in mathematics toward determining which affec-
tive characteristics affect student performance and growth.

> Many psychological tests have been used in research aimed at
> uncovering important relationships between cognitive and
> affective variables. To date the findings have been minimal
> . . . in spite of the fact that there is almost universal agree-
> ment that affective variables play an essential role in the
> learning of mathematics. New ideas are badly needed for appro-
> priate and sensitive measures of the affective component in
> mathematics education. (NACOME, 1975, p. 126)

The questions of interest in this study center around certain
affective characteristics and their relationship to student preference
to do certain nonroutine mathematics problems. Given that there are
certain definable and distinguishable personality preferences, as
defined by Jung (1971), does a subject exhibiting a particular per-
sonality style prefer to do certain types of mathematical problems?
Of particular interest is the question of what kinds of mathematical
problems a subject would prefer to solve.

There have been many attempts to determine whether there are any
affective characteristics which correlate with success in mathematics
and mathematical problem solving. Montor (1973) attempted to determine

whether there were any differences in brain wave patterns when students were at rest, solving problems, or under stress. While she found differences in the brain wave patterns among the different kinds of activities, her data failed to support the thesis that there would be a difference in brain wave patterns between students with high and low grade point averages during these activities. Ruble and Nakamura (1972) investigated four aspects of outerdirectedness with respect to problem-solving tasks: developmental trends, effects of different types of reinforcement, effects of task difficulty, and pride of accomplishment. They found that outerdirectedness decreased with age and increased when the task was described as difficult and was associated with pride ratings of children. Spencer (1957) found that high-anxiety subjects responded less accurately than low-anxiety subjects to problems containing value statements when the problems were presented in an emotional context. Trimmer (1974), in his review of research relating problem solving and mathematical achievement to psychological variables, listed 10 traits that have been studied: attitude, debilitating impulsive/reflexive thinking, self-concept, orderliness, set, confidence, motivation, interest, perserverance, and patience. In summary, he noted that "the impulsive state is probably useful in producing hypotheses and the reflective state is probably best for evaluating hypotheses. When the going gets tough it is concentration and persistence that pay off. It is then that confidence, lack of anxiety, lack of rigidity, flexibility, and an ability to cope with uncertainty help" (p. 22).

Instrumentation

To determine any relationship which may exist between personality characteristics and the preference for one mathematical problem or type of problem over another, two instruments were used. The first was a set of 24 nonroutine mathematics problems (Appendix A). Developed as part of a National Science Foundation research project (Kantowski, 1980), the four types of problems used were number theoretic problems that could be solved by using an inductive strategy, number theoretic problems that could be solved using factors and other properties of the quantities involved, logic problems, and geometry problems. These categories denote the mathematical content of the problems. In addition, a post-hoc analysis showed that the problems also varied in terms of their structure. The three types of structure variables included all of the content types except the problems in geometry. An example of a problem with a different content and structure type is problem 24, which asks the question, "How many lines may be drawn joining 18 points on a circle?" This is an example of a problem with a geometry content but whose mathematical structure would lead to an inductive solution.

The second instrument used was the Myers-Briggs Type Indicator, Form F (Myers, 1962). This is a self-reporting personality preference inventory using a modified version of the dichotomous scales suggested by Jung. The four scales are

Index	Preference
E-I	Extraversion or Introversion
S-N	Sensing or Intuition
T-F	Thinking or Feeling
J-P	Judgment or Perception

Persons who exhibit a preference for extraversion will think, feel, act, and actually live in a way that is directly correlated with the objective conditions (people and things) and their demands (Jung, 1971). Introverts, on the other hand, while aware of external conditions (their environment), prefer the selection of subjective determinants (concepts and ideas) with respect to the direction of their judgment and perception.

The S-N index is designed to reflect which of the two perceiving strategies individuals prefer, namely, sensing or intuition. Persons who prefer to use sensing are made aware of things directly through one or more of the five senses. Persons who prefer to use intuition rely primarily on the less obvious process of indirect perception by way of the subconscious. There are implied ideas or associations which the unconscious tacks on the thing perceived. This explains why there is often no obvious relationship between the object and intuitive types' perceptions of that object.

The T-F index reflects whether an individual basically makes his or her judgments in a thinking or feeling way. If a person prefers thinking, then his or her judgments will be in terms of true or false or other impersonal measures. He or she will perform best when organizing facts and ideas. A person preferring to make judgments using feeling will discriminate between stimuli in terms of valued and not valued criteria, or other personal criteria, and will perform best when dealing with human relationships.

The J-P index is designed to reflect whether individuals prefer to use their judging process, i.e., either thinking or feeling, or

their perceptive process, which is either sensing or intuition, in dealing with their environment. Persons who prefer to use their judging process will want to come to conclusions, to get things settled. They will want to run their own lives. Persons who prefer to use their perceptive process will want to wait to see if there may be more information available, which either their sensing or intuition, whichever is dominant, will gather in for them. They will be looking for other factors to consider and will avoid irrevocable choices.

Questions of Interest

The five questions of interest studied were

1. Given a set of nonroutine mathematics problems, do students sort the problems in similar order?

2. During the sorting-by-preference activity, do students sort the problems according to the mathematical content of each problem?

3. Do students with the same Myers-Briggs type element (E, I, S, N, T, F, J, P) order the problems in the same way? Is there an agreement as to the ordering of the problems for each of the eight personality characteristics?

4. Is there a difference in the way persons with opposite type elements order the problems (E vs. I, S vs. N, T vs. F, J vs. P)?

5. Is there any relationship between individual students on the sorting by preference to do the problem?

Statistical Procedures

In this study, each student was asked to sort a group of 24 nonroutine mathematics problems. A ranking was then assigned each problem with 24 being the most preferred and 1 being assigned to the least preferred problem. Since these rankings are ordinal, two nonparametric statistical procedures were employed: the Spearman rank correlation coefficient (rho) which is useful in predicting intergroup similarities, and the Friedman rank sums test, which can be used to predict intragroup similarities.

Importance of the Study

A review of the literature revealed an extensive body of research concerning the problem-solving process. Most of this research examined those characteristics of the problem which affect student performance. Other research examined those characteristics of the student which may affect the problem-solving process. The purpose of this study is to determine whether there are characteristics of a student's personality which influence his or her preference to want to do certain nonroutine mathematics problems. If a relationship between personality and preference does exist, then it would be possible to design an educational program which would use preference as its foundation. What is being examined is the possible predisposition of a student to want to do one type of problem over another.

Description of the Population

Subjects in this study were 18 students from Bradford High School, in Starke, Florida. These students, in grades 9 through 12, were enrolled in a special course in problem solving which was designed

to teach heuristic strategies for solving mathematical problems. Twelve of the students were enrolled in the school's program for the gifted, while the remaining six were considered high achievers by their mathematics instructors. All students had completed at least Algebra II with the exception of one 9th grader and one 10th grader who were enrolled in it when the study was conducted. A complete listing of students with respect to grade, mathematics courses taken, and IQ is included in Appendix C.

Organization of the Study

In this first chapter the framework of the study has been developed by presenting the background, statement of the problem, instrumentation, questions of interest, statistical procedures, importance of the study, and description of the population. Chapter Two contains a theoretical base along with a supporting review of the literature. Chapter Three presents the methods and procedures used in the study. Chapter Four contains an analysis of the data, and Chapter Five contains the summary and conclusions, as well as implications for future research.

As indicated in Chapter One, the purpose of this study is to
investigate whether there is a relationship between students' per-
sonality preference and preference for trying certain nonroutine
mathematics problems. Related to this question of personality prefer-
ence and nonroutine mathematics problems is the need also to examine
those factors which have been identified by others as contributing to
variances in students' perceptions of mathematical problems. In this
study, factors—that is, those factors which can be attributed to
the task itself—are classified as task variables. In the next sec-
tion a theoretical base for task variables is developed. Included
in that section is a review of the literature which supports this base.
Two recent dissertations by Chartoff (1976) and Silver (1977) are
investigated in the next section. The last part of the chapter con-
tains a review of the literature as it pertains to the Myers-Briggs
Type Indicator.

Theoretical Base

The model for this dissertation is based on the works of George
Polya and Newell and Simon (see Figure 2-1). Polya (1957) is correctly
credited with being the father of modern heuristic reasoning. He
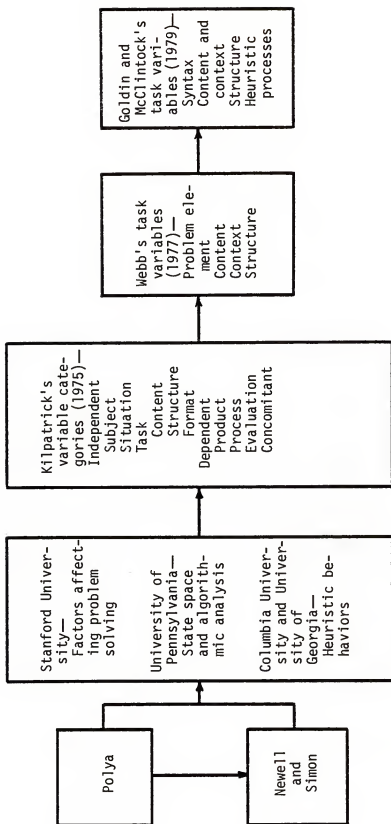codified and refined this process so that others could build upon its

Figure 2-1. Evolution of task variables as a factor in the analysis of the problem-solving process.

foundation. He developed a four-step plan for solving problems (see Table 2-1). It is perhaps the most often quoted and researched strategy developed to help students solve problems. With respect to task variable, Kulm (1979) noted that the most intriguing part of Polya's heuristic process "is the advice to find a 'related problem'" (p. 8). According to Polya, there are three ways in which problems may be related: by analogy, by specialization, and by generalization. Kulm also mentioned that related problems may be found by decomposing and recombining problems. Problems may be related in other ways, such as through syntax or context, terms to be defined later in this chapter. The relatedness of problems is examined in the research in terms of the differences perceived by students as they sort the 24 problems used in this study, from the one they would most like to solve to the one they would least like to solve. One specific question to be investigated is whether students will perceive similarities in problems based on the content and structure of the problem.

In searching for a way to describe the problem-solving process, Newell and Simon (1972) provided a theoretical framework where the solution of a problem is derived from the successful outcome of a search process. They characterized behavior during the search process (problem solving) as an information processing system (IPS). According to Heller and Greeno (1978), important elements of the IPS are

1. Short- and long-term memory

2. Pattern recognition

3. Comparison processes

4. Symbol manipulation.

Table 2-1

Polya's "How To Solve It" List

First. You have to understand the problem.	**Understanding the Problem** What is the unknown? What are the data? What is the condition? Is it possible to satisfy the condition? Is the condition sufficient to determine the unknown? Or is it insufficient? Or redundant? Or contradictory? Draw a figure. Introduce suitable notation. Separate the various parts of the condition. Can you write them down?
Second. Find the connection between the data and the unknown. You may be obliged to consider auxiliary problems if an immediate connection cannot be found. You should obtain eventually a plan of the solution.	**Devising a Plan** Have you seen it before? Or have you seen the same problem in slightly different form? Do you know a related problem? Do you know a theorem that could be useful? Look at the unknown! And try to think of a familiar problem having the same or a similar unknown. Here is a problem related to yours and solved before. Could you use it? Could you use its result? Could you use its method? Should you introduce some auxiliary element in order to make its use possible? Could you restate the problem? Could you restate it still differently? Go back to definitions. If you cannot solve the proposed problem try to solve first some related problem. Could you imagine a more accessible related problem? A more general problem? A more special problem? An analogous problem? Could you solve a part of the problem? Keep only a part of the condition, drop the other part; how far is the unknown then determined, how can it vary? Could you derive something useful from the data? Could you think of other

Table 2-1 (continued)

	data appropriate to determine the unknown? Could you change the unknown or the data, or both if necessary, so that the new unknown and the new data are nearer to each other? Did you use all the data? Did you use the whole condition? Have you taken into account all essential notions involved in the problem?

Carrying Out the Plan

Third.
Carry out your plan.

Carrying out your plan of solution, check each step. Can you see clearly that the step is correct? Can you prove that it is correct?

Looking Back

Fourth.
Examine the solution obtained.

Can you check the result? Can you check the argument?
Can you derive the result differently? Can you see it at a glance?
Can you use the result, or the method, for some other problem?

Source: Polya, 1957.

Heller and Greeno reiterated the following characteristics which Newell and Simon described as invariant over task and problem solver:

1. All problem solving takes place in a problem-solving space which contains not only the actual solutions, but also all possible solutions.

2. To solve the problem, the problem solver (computer or human) must search the problem space. This search occurs as the problem solver considers alternate paths from the initial problem state to the goal state.

3. The problem solver uses a means-ends analysis. He works to select appropriate means (operators) to achieve the desired ends (goals). His choices are delimited by an evaluation of his present state compared to the goal state.

4. To counter the one-step-at-a-time limitation of means-ends analysis, the problem solver employs a general strategy derived from an appropriate planning process.

From this framework, Goldin and McClintock (1979) noted that three main avenues of investigation developed:

1. Factors affecting problem difficulty were investigated by Suppes, Jerman, and others at Stanford University during the 1960s. This occurred during the development of the SMSG materials but was undoubedly influenced by the work of Polya, who was also at Stanford.

2. State space and algorithmic analysis of problem structure were investigated by Goldin and his graduate students at the University of Pennsylvania during the early to mid-seventies.

3. The description and analysis of heuristic behavior was investigated by Kilpatrick and his graduate students at Columbia University. Independent analysis was also being conducted by Wilson, Hatfield, and Kantowski at the University of Georgia.

In May, 1975, many of these researchers met at the University of Georgia to discuss problems of common concern. The ideas expressed in one of the papers presented at the meeting provide the next cornerstone for the theoretical base of this investigation. In his paper, "Variables and Methodologies in Research on Problem Solving," Kilpatrick (1975) identified seven variables related to the problem-solving process, three independent variables and four dependent variables. He identified the independent variables as subject, situation, and task. A variable is independent if it is used in making predictions. The four dependent variables are product, process, evaluation, and concomitant. A variable is dependent if it refers to the behavior being predicted (see Table 2-2).

Kilpatrick (1975) divided subject variables into organismic, trait, and instructional history. Organismic variables include such factors as age, sex, race, and social class. Trait variables include abilities, attitudes, interests, values, and other personality variables relating to perceptual style, congnitive style, self-concept, persistence, anxiety, and need for achievement. Kilpatrick's (1975) concept of perceptual style will be one focus of this study. A question of interest will be considering a student's personality preference—will students with certain personality styles tend to order their

Table 2-2

Kilpatrick's Categories of Problem-Solving Research Variables

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I. Independent variables
    A. Subject variables
        1. organismic variables
        2. trait variables
        3. instructional history variables
    B. Task variables
        1. context variables
        2. structure variables
        3. format variables
    C. Situation variables
        1. physical setting
        2. psychological setting
II. Dependent variables
    A. Product variables
    B. Process variables
    C. Evaluation variables
    D. Concomitant variables

Source: Kilpatrick, 1975.

preference to do certain nonroutine mathematics problems in predict-
able ways.

The third subject variable in Kilpatrick's framework, instruc-
tional history, involves those elements that the subject has learned.
The ability to solve problems is totally dependent on what information,
algorithms, and devices the solver has in his repertoire of mathematical
tools. Instructional history is the underpinning of many of the task
variables.

Kilpatrick viewed situation variables as those possible varia-
tions that may occur in a subject's perception of the task. Both physi-
cal and psychological factors may affect this variable. All factors
which are external to the learner's knowledge and the problem task are
subsumed under this heading. Such diverse factors as room lighting,
noise level, and reason for attempting the task are all situation
variables. Kilpatrick (1975) notes that "situation variables them-
selves are of relatively little interest to mathematics educators,
although they may interest the social or educational psychologist.
They are nuisance variables—since they will not go away, one can only
hope they will not make much difference. Unfortunately, they may"
(p. 5).

Kilpatrick divided the third independent variable, task vari-
ables, into three parts: context, structure, and format variables.
Webb (1977) further refined this network into four main divisions of
task variables: problem element, content, context, and structure,
with Kilpatrick's format variable being subsumed under the context
variable. Webb (1979) later subsumed problem element as part of the
content variable.

Building upon the above works, Goldin and McClintock (1979) further refined the task variable category into four subsections: problem syntax, the mathematical content and nonmathematical context of the problem, problem structure, and the heuristic processes evoked from the problem (see Table 2-3).

Syntax variables (problem syntax) are those factors concerning the surface structure of the problem. Barnett (1979) defined syntax variables as "those variables which account for the arrangement of and the relationship among words, phrases, and symbols in problem statements" (p. 23). He noted that "unlike many task variables which depend on an interpretation of the processes to be used . . . the definition and quantification of syntax variables can be derived almost directly from the problem statement" (p. 24).

In his review of the research, Barnett found five major categories of syntax variables which affected problem-solving performance. First, the length of words, phrases, and sentences tends to affect problem-solving performance as increased load demands are placed on short- and long-term memory and the ability to reorganize data. Second, grammatical structure variables such as number of main clauses, number of subordinate clauses, and the number of prepositional phrases affect problem difficulty because of the increased need for reorganization. Third, the importance of variables describing numerals and mathematical symbols will depend on the subject's familiarity with the form of the problem presented and its relationship to the form required for a mathematical equation. Fourth, the length of the question sentence and its location in the problem statement are variables that have been

Table 2-3

Goldin and McClintock's Task Variables

 I. Problem syntax
    A. length of words, phrases and sentences
    B. grammatical structure
    C. numerals and mathematical symbols
    D. length of question sentence
    E. sequencing of data
 II. Content and context variables
    A. content variables
       1. mathematical topic
       2. field of application
       3. semantic content
       4. problem elements
    B. content variables
       1. problem embodiment
       2. verbal setting
       3. information format
III. Problem structure
 IV. Heuristic processes

Source: Goldin and McClintock, 1979.

investigated with varying results. These results have failed to estab-
lish clearly whether a question sentence used at the beginning of the
problem acts as an advanced organizer or whether its "distance" from
the end of the statement causes the student to lose sight of the task.
The fifth syntax variable identified involves the sequencing of data in
the problem statement. Studies by Loftus (1970) and Segalla (1973)
indicated that problems with data presented in the same order as they
will be used to reach a solution tend to be easier than those in which
the data are out of order.

The second of the four major task variables categories is divided
into the mathematical content and nonmathematical context of the prob-
lem. Content, according to Webb (1979), is the mathematical substance
of the task. His four content variable categories are mathematical
topic, field of application, semantic content, and problem elements.
Mathematical topic variables can be divided into two parts, subject
area and problem type. Subject area refers to the specific field of
mathematics, such as algebra and geometry. Problem type refers to
similarities in the problem statement. Typical problem type classifi-
cations include age and rate problems, which may be solved in a
variety of ways, depending on the experience of the problem solver.

The second of the content variables, field of application, refers
to problems that are derived from real-life situations. Webb (1979)
makes the distinction between genuine applied problems and those that
are "merely embedded in a story" (p. 84), with the latter type essen-
tially providing a context for the task. The third content variable,
semantic content, includes those factors related to the mathematical

vocabulary of the problem statement. Kane (1968, 1970) pointed out that mathematical English and ordinary English differ on at least four factors:

1. The level of redundancy of words
2. The unique denotation of names of mathematical objects
3. The importance of adjectives
4. The lower flexibility of grammar and syntax.

Studies of semantic content have tended to deal with training in mathematical vocabulary, the use of verbal cues both in training and in determining problem difficulty. Barnett (1979) listed the use of key words in the problem-solving process as a semantic variable. Kilpatrick (1979), however, argued that key words should not be considered to refer to content but to syntax. He noted that only problems with the same relationship between data and the unknown, despite any representational differences, should be considered the same with respect to the content variable which determines the structure of the problem.

The last content variable, problem elements, was defined by Wicklegren (1974) as containing "three types of information: information concerning givens (given expressions), information concerning operations that transform one or more expressions into one or more new expressions, and information concerning goals (goal expressions)" (p. 10). To Wicklegren, these three types of information are what constitute a problem, and, while not all elements are immediately apparent in all problems, especially those in the real world, the solution will contain all necessary elements and the appropriate solution path to utilize the elements.

In discussing the nonmathematical context of a task, Webb (1977, 1979) noted that context refers to circumstances, surroundings, formats, and instructions that are included as part of the task and which influence the understanding of the task. Context variables fall into three broad categories: problem embodiment, verbal setting, and information format. The use of manipulatives, pictures, symbols, or verbal stories can all be classified as problem embodiment. Verbal setting variables are those which, if they change, may affect a student's perception of the problem. Dichotomies which have been investigated include familiar versus unfamiliar setting, concrete versus abstract, and factual versus hypothetical. Webb (1979) noted that "the preponderance of evidence shows that most of these variables do not greatly affect problem difficulty when other variables are controlled" (p. 100). In particular, he pointed to a study by Brownell and Stretch (1931) which suggested that problems involving unfamiliar settings were not more difficult for children except under a limited set of conditions. Of interest to those teaching mathematics is a study by Cohen (1976) which investigated outdoor, computational, and scientific settings with regard to problem difficulty. In an investigation of 200 eighth graders, he concluded that area of student interest alone was not sufficient to predict success on any particular type of problem.

The third of the context variables, information format variables, includes those factors concerned with the presentation of the problem with relation to the information given. The partitioning of information by either giving all the information at once or presenting it piecemeal to allow for internal processing will certainly affect the

problem solution. The use of hints as demonstrated by Krutetskii (1976) and other Russian mathematics educators would also fall into this category.

Unlike other task variables which may be described in terms external to the problem statement, problem structure, the third task variable, may only be obtained by some analysis of the problem. We may think of the problem structure as the path the problem solver traverses as he goes from the initial state to the goal state. There may be many paths for each problem statement, and along each path may be many subgoals which must be completed to proceed along the solution path. Goldin (1979) used Nillson's definition to describe a state space for a problem as "a set of distinguishable problem configurations, called states, together with the permitted steps from one state to another, called moves" (p. 115). Each problem will have an initial state and one or more goal states.

The fourth and last task variable in our model is the heuristic process variable. As was stated previously, Polya is credited with being the father of modern heuristics. Polya (1957) viewed modern heuristics as endeavoring to examine those processes used in solving problems. He noted that "experience in solving problems and experience in watching other people solving problems must be the basis upon which heuristics is built" (p. 130). While this would seem to indicate that heuristic reasoning is a characteristic of the problem solver, McClintock (1979) pointed out that heuristic processes may be viewed as inherent in the mathematical problems, that it is the task which interacts with the mental operations of the problem solver to evoke the appropriate heuristic processes.

Recent Studies

Two recent dissertations have focused on combinations of factors which affect the problem-solving process. Chartoff (1976) used a multi-dimensional scaling procedure, INDSCAL, to analyze subject dissimilarity data with respect to classification according to type of problem. Students were given several pairs of problems to rate on a scale from very similar to very dissimilar. They were asked to rate the problems both before and after seeing the solutions.

In analyzing the data, Chartoff found that students sorted the problems into four broad categories which he called dimensions. He gave the following interpretation to these dimensions:

> Dimension one—various equivalent phrases are used to describe this dimension, including recognition of the Polya variations, classification according to type, and recognition of how the problems are solved.
> Dimension two—contextual setting with emphasis of context over content. At times this dimension takes on a concrete-abstract form.
> Dimension three—this is a general dimension where problems are compared to the generalization of the respective types. After the solutions have been seen, this dimension often changes to a solvable-unsolvable classification, where unsolvable takes broad interpretation.
> Dimension four—classification based on the question or goal of the problem. (pp. 51-52)

The analysis of these dimensions was accomplished by a post-hoc investigation of the problem space, which primarily took into account the built-in variables.

The dimensions, as described by Chartoff, can be related to the framework of this chapter in the following ways. Dimension one, recognition of the Polya variations, is a very gross category covering almost all the task variables and heuristic behaviors. Dimension two, contextual

setting, is the most easily identifiable of the task variables in our framework. Dimension three, comparison with generic problems, would, perhaps, best be compared to the problem structure described by Webb (1977). Dimension four, classification based on the problem question, is similar to the goal-oriented heuristic investigated by a number of researchers. McClintock (1977) noted that the importance of this factor has been recognized by many leaders in the problem-solving field. Wicklegren (1974), in discussion his strategies of state evaluation and hill climbing, defined the first as evaluating all the states including the goal state and defined the latter as choosing activities which will achieve a state closer to the goal.

Current research on goal-oriented heuristics has been characterized by the work of Kantowski (1974). She found that when goal-oriented heuristics were used, the solution tended to be more efficient. She further found more regular patterns of analysis and synthesis where goal-oriented heuristics were used than where they were not.

Silver (1977) also studied the relationships students perceive in mathematical word problems. Six exercises were developed to determine different aspects of student perception. In addition, data from five standardized instruments were gathered in an attempt to determine any relationship between students' performance on the ability tests and their perception of problem relatedness.

The problems were designed to be similar on two dimensions, mathematical structure and contextual detail. However, initial research indicated two additional dimensions which students used in classifying the problems. Silver (1977) found that they often grouped problems on

the basis of the question being asked or on the basis of the quantities involved, such as weights or distance. These became the third and fourth dimensions, respectively, of his model. The first of these is interesting since it also showed up in Chartoff's (1976) work and further supports the case by Kantowski (1974) concerning goal-oriented heuristics.

The fourth dimension appears to be related to the context dimension, but recognizing the fact that problems with the same quantities are often solved in the same way, i.e., have the same mathematical structure, Silver named this the pseudostructure. Analysis of his data showed that this dimension was indeed positively correlated with context but was significantly negatively correlated with structure. It is, therefore, likely that the pseudostructure dimension could be subsumed under the context dimension so that only three dimensions remain.

Research Investigating the Myers-Briggs Type Indicator

The Myers-Briggs Type Indicator (MBTI) is a self-reporting psychological inventory based upon the works of Carl Jung. The MBTI was developed over a twenty-year period by Isabel Briggs Myers. Her purpose was to develop an instrument which could discriminate among the different types which were postulated by Jung. In 1962, the Educational Testing Service published the instrument, and in the ensuing twenty years researchers have used it as a tool for uncovering possible relationships between individuals' personality preference and their environment.

There has been an extensive body of research concerning personality, as measured by the MBTI, and career choice. Health-related professions (McCaulley, 1978), science (McCaulley, 1976a), and engineering (McCaulley, 1976b) have received much attention. There appear to be two

purposes for using the MBTI as one element in developing a clear picture
of career choices. First, it can be used as a counseling tool, and
second, it can be used to determine the most appropriate type of instruc-
tion for the individual (Lawrence, 1979).

There has been a considerable amount of research concerning per-
sonality type and student ability. Hoffman and Betkouski (1981), in
their summary of research applications using the MBTI, found several
studies which compared personality type with other measures of student
performance. Ross (1966), using the data from 319 male and 252 female
high school students, intercorrelated the four MBTI scales (E-I, S-N,
T-F, and J-P) with a battery of 32 tests including 15 ability tests.
His results indicated that all four scales reflected some surface charac-
teristics other than the typological differences for which they were
constructed. Of the four scales, Ross found that the S-N dichotomy
showed high loading for both males and females on the general ability
factor. He stated, "It is clear that the S-N scale is a very useful
and unusual one since it gives information about both ability and atti-
tude, using only self-report questions" (p. 14).

In a 1974 study, McCaulley and Natter noted that "the most con-
sistent finding in Myers' research and in the studies of others . . . is
that intuitive types average higher scores on aptitude measures than
sensing types" (p. 117). In the same study, they reported on a research
project conducted at the Florida State University Developmental Research
School which investigated the relationship between the dichotomous
scales of the MBTI and certain measures of academic performance. These
measures included grade point averages and results on the Florida
Eighth Grade Test, the Florida Ninth Grade Test, the Florida Twelfth

Grade Test, the PSAT, the Gates Reading Comprehension Test, the Armed Services Vocational Appitude Battery, and the California Test of Mental Maturity. (A listing of these instruments is contained in Table 2-4).

With respect to reading, McCaulley and Natter (1974) noted that intuitive types scored significantly higher on 10 of 11 measures relating to reading ability and on 3 social studies measures requiring reading skills. On 5 of 8 mathematics measures and 2 of 3 science measures, intuitives significantly outscored sensing types. The difference between significance and nonsignificance on these measures can be interpreted in terms of problem solving versus computational activities. This would indicate, that, while intuitive types do better on problem-solving tasks, they are equal to sensing types on computational tasks. The study further showed that intuitive types scored significantly higher than sensing types on measures of academic aptitude and study skills.

With respect to the other indices, McCaulley and Natter (1974) noted that in academic measures thinking types scored significantly higher than feeling types in electrical, mechanical, motor mechanical, and technical areas. Extraverts scored significantly lower than introverts on several academic measures including aptitude, reading, and mathematics. Perceptive types outscored judging types in every measure of academic achievement except for grades in school.

Table 2-4

Academic Measures Used in a Study Conducted at the Florida State
University Developmental School

I. Grades in school
   A. Overall average
   B. English
   C. Mathematics
   D. Science
   E. Social studies
II. Florida Eighth Grade Test
   A. Vocabulary
   B. Comprehension
   C. Computation
   D. Problem solving
   E. Everyday living
      mathematics
   F. Everyday living
      reading
   G. Study skills
III. Florida Ninth Grade Test
   A. Verbal
   B. Quantitative
   C. Social studies
   D. English
   E. Mathematics I
   F. Mathematics II
   G. Science

IV. Florida Twelfth Grade Test
   A. Aptitude
   B. English
   C. Social studies
   D. Reading index
   E. Natural science
   F. Mathematics
V. Preliminary Scholastic Aptitude
   Test
   A. Verbal
   B. Mathematics
VI. Gales Reading Test
   A. Vocabulary
   B. Comprehension
VII. Armed Services Vocational Aptitude
   Battery
   A. Electrical
   B. Motor mechanical
   C. General mechanical
   D. Clerical/administrative
   E. General technical
VIII. California Test of Mental Maturity

Source: McCaulley and Natter, 1974.

CHAPTER THREE
METHODOLOGY

This chapter contains a description of the population, instrumentation, design, and administrative procedures used in the study.

Population

Eighteen students in grades 9 through 12 at Bradford High School, in Starke, Florida, participated in the study. Of the 18 students, 12 had been classified as educationally gifted. The major criterion for being certified gifted in the state of Florida is a derived score of at least 130 (2 standard deviations above the mean) on a recognized IQ test. Scores for this group ranged from 130 to 146. The remaining 6 students were chosen because

1. Their IQ scores were close to the cut-off point (130) for gifted students or

2. They had performed well in previous mathematics classes. Student IQ and grade in school, as well as previous courses taken, are listed in Appendix C.

During the 1977-1978 school year, these students were placed in a special mathematics class on problem solving. The purpose of the class was to provide training in the use of heuristic strategies employed in the problem-solving process. The framework of the program was to employ those strategies suggested by Polya (1957) for solving a variety of nonroutine mathematics problems. This framework is similar to

those strategies employed by Wilson (1967), Kilpatrick (1967), Kantowski (1974), and others. Specifically, the strategies are

1. Understanding the problem

2. Devising a plan

3. Carrying out the plan

4. Looking back.

For each of these strategies there is a series of questions which the student should utilize in attempting to obtain a solution to the problem. For example, under the strategy of devising a plan, the student should ask, "Do I know a related problem?" This type of questioning was stressed throughout the course.

Instruction in these heuristic processes was provided as part of a research project in problem solving conducted by Kantowski. This research, while not related to the present study, investigated the use of heuristic strategies in solving a variety of problems. As Kantowski (1980) noted,

> The instruction phase was included because it was found (Kilpatrick, 1967; Kantowski, 1974), as suggested by Newell and Simon, that without some instruction students generally revert to trial and error in solving problems or do not attempt to solve them at all. . . . Therefore instruction in the use was given and evidence of the use of these heuristic processes and the relationship of their success in problem solving was sought. (pp. 13-14)

Kantowski's research was divided into three phases. In the first phase, students were given four nonroutine problems of the type used in this study and were asked to solve them while thinking aloud. Both the verbal and written protocols were analyzed to determine what, if any, heuristic strategies were used and what solution paths were followed in the solution process. The second phase of the research project included

22 instructional sessions designed to introduce subjects to Polya's four phases previously mentioned. During instruction, emphasis was placed on (1) what was meant by "understanding the problem," (2) several types of plans or strategies that would be useful in solving a problem, (3) the use of related problems, and (4) the importance of the looking-back strategy both as a tool for checking a solution and as a means for finding different and more efficient solutions.

The use of the examples was the primary means of introducing both new material and heuristic strategies. Problems were presented and the students were carefully guided into appropriate actions. The heuristic strategies were consistently repeated during the problem-solving process so that the subjects might see the value of using them in attaining a solution. Each of the problem types used in the current study was presented and the students were given experience in solving them. Concurrently with the instruction, the students were given 10 sets of four problems and were asked to solve them while thinking aloud.

At the conclusion of the training and problem-solving sessions, the students were given the set of 24 problems and asked to sort them from the one they would most like to do to the one they would least like to do. Each student was placed in a quiet office with no distractions and allowed as much time as he or she needed to complete the assignment.

Myers-Briggs Type Indicator

The Myers-Briggs Type Indicator aims to ascertain subjects' basic personality preferences with respect to their judgment and perception. This self-reporting inventory uses a modified version of the dichoto-mous scales suggested by Jung. The four scales are extraversion

(E)-introversion (I), sensing (S)-intuition (N), thinking (T)-feeling (F), and judgment (J)-perception (P). The individual's type is denoted by use of a 4-letter combination, such as INTP or ESTJ, with each letter representing that individual's preference on each of the four scales. The personality type of each student is included as part of Appendix C, while Table 3-1 contains the distribution of each type as well as the number of students with a preference for each type element. Each student is represented four times in this study, once for each type element.

Problem Set

The problem set consisted of 24 nonroutine mathematics problems. A problem is here defined to be a question to which the student does not know an algorithm for its solution. The student may or may not know the requisite mathematical properties to derive the solution. If he or she has the appropriate knowledge, then a plan of attack, as suggested by Wicklegren (1974), or the implementation of the heuristic strategies, as suggested by Polya (1957), should aid in the solution of the problem. A mathematical question to which the student knows an appropriate algorithm is merely an exercise.

The problem set used in this study (Appendix A) was developed as part of a National Science Foundation research project. The 24 problems varied in both content and structure. The 4 content types were number theoretic problems that could be solved using an inductive strategy, number theoretic problems that could be solved using factors of other properties of the quantities involved, logic problems, and geometry problems. The 3 structure types were number theoretic problems that could be solved using an inductive strategy, number theoretic problems that could be solved using factors or other properties of the quantities involved, and

Table 3-1

Distribution of Students by Personality Preference Using the
Myers-Briggs Type Indicator
(N = 18)

SENSING TYPES		INTUITIVE TYPES	
with THINKING	with FEELING	with FEELING	with THINKING
ISTJ	*ISFJ*	*INFJ*	*INTJ*
N = 0	N = 1	N = 0	N = 1
% = 0	% = 5.5	% = 0	% = 5.5
ISTP	*ISFP*	*INFP*	*INTP*
N = 0	N = 2	N = 6	N = 1
% = 0	% = 11.1	% = 33.3	% = 5.5
ESTP	*ESFP*	*ENFP*	*ENTP*
N = 0	N = 1	N = 2	N = 0
% = 0	% = 5.5	% = 11.1	% = 0
ESTJ	*ESFJ*	*ENFJ*	*ENTJ*
N = 1	N = 1	N = 2	N = 0
% = 5.5	% = 5.5	% = 11.1	% = 0

JUDGING — INTROVERTS — PERCEPTIVE — PERCEPTIVE — EXTRAVERTS — JUDGING

	Number	Percent
E	7	38.8
I	11	61.1
S	7	38.8
N	11	61.1
T	3	16.6
F	15	83.3
J	6	33.3
P	12	66.6
I J	2	11.1
I P	9	50.0
E P	3	16.6
E J	4	22.2
S T	1	5.5
S F	6	33.3
N F	9	50.0
N T	2	11.1
S J	3	16.6
S P	4	22.2
N P	8	44.4
N J	3	16.6
T J	2	11.1
T P	1	5.5
F P	11	61.1
F J	4	22.2
I N	7	38.8
E N	4	22.2
I S	4	22.2
E S	3	16.6

logic problems. Problems were either the same with respect to content and structure or were a combination of types. Table 3-2 lists the content and structure type for each problem.

Three months after the final instructional unit, the students were presented with the card-sorting task. Each of the 24 problems was printed on a separate 5 x 8 index card and the required task was for the student to sort (rank) the set of problems from the one he or she would most like to do to the one he or she would least like to do. The students were not asked to solve the problems. (The instructions page is presented in Appendix B.) Three weeks after the card-sorting task was completed, the Myers-Briggs Type Indicator was administered to all students.

Table 3-2

Content and Structure of Problems

Problem	Content	Structure	Combination
1	Inductive	Inductive	Pure[a]
2	Geometry	Inductive	Composite
3	Logic	Logic	Pure
4	Logic	Factor	Composite
5	Geometry	Inductive	Composite
6	Factor	Factor	Pure
7	Logic	Inductive	Composite
8	Factor	Factor	Pure
9	Geometry	Inductive	Composite
10	Logic	Factor	Composite
11	Factor	Inductive	Composite
12	Logic	Inductive	Composite
13	Logic	Factor	Composite
14	Geometry	Inductive	Composite
15	Geometry	Inductive	Composite
16	Logic	Logic	Pure
17	Inductive	Inductive	Pure
18	Factor	Factor	Pure
19	Geometry	Inductive	Composite
20	Geometry	Inductive	Composite
21	Logic	Logic	Pure
22	Inductive	Inductive	Pure
23	Logic	Factor	Combination
24	Geometry	Inductive	Combination

[a]Pure denotes same content and structure.

This chapter contains the results of the study and a statistical analysis of the data. Since the data collected were ordinal (ranked), two nonparametric statistical procedures were used, the Friedman rank sums test and the Spearman rank correlation coefficient.

Analysis of the Data

The data for this study are the result of asking 18 bright high school students to sort a set of 24 nonroutine mathematics problems from the one they would most like to do to the one they would least like to do. This sorting task came at the conclusion of a 22-lesson program designed to teach heuristic strategies. It can, therefore, be assumed that each student had seen several examples of each type of problem used in this study.

The results of the ordering are presented in Table 4-1, with 24 being associated with the student's first choice and 1 being associated with that problem that he would least like to do. The answers to all questions proposed in this study come from an analysis of this set of data or from an examination of the student school records.

Questions of Interest

Question One

The first question investigated in this study was: Given a set of nonroutine mathematics problems, is there a regularity in the way

Table 4-1

Results of Sorting Task

Student	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
1	24	6	22	14	13	2	15	9	4	23	5	8	18	21	1	20	17	11	12	3	19	7	10	16
2	17	12	21	24	15	5	11	14	9	8	16	13	20	3	10	22	19	7	6	4	23	18	1	2
3	24	19	13	3	18	1	4	8	2	23	17	22	12	16	6	11	21	10	5	15	9	20	7	14
4	4	7	20	16	8	3	11	21	6	15	17	9	18	1	13	19	10	23	5	2	24	14	22	12
5	17	8	22	23	11	7	9	16	4	24	13	12	19	2	21	10	18	5	1	6	20	14	15	3
6	21	11	24	7	12	13	19	2	8	4	14	18	22	15	9	1	3	20	17	10	23	16	5	6
7	6	5	24	11	9	18	4	21	17	12	21	7	13	16	11	2	22	7	15	8	20	14	18	1
8	10	5	23	19	14	6	5	21	15	12	21	9	19	1	4	24	9	23	16	4	8	22	11	3
9	15	8	21	16	10	4	11	16	14	17	12	2	22	5	9	24	3	20	19	11	23	6	18	7
10	14	6	19	24	7	13	10	13	17	11	21	8	21	1	11	22	15	22	15	11	2	16	12	3
11	24	5	4	17	19	1	21	20	8	4	21	8	11	18	18	1	22	5	20	6	17	19	8	15
12	24	22	13	16	14	18	11	7	16	2	4	5	13	16	6	18	3	19	12	9	20	8	23	3
13	9	14	8	9	13	6	5	12	2	17	17	7	23	13	2	22	4	21	10	3	20	17	19	1
14	18	8	24	17	9	10	16	18	16	24	24	14	23	2	11	23	13	21	9	7	24	17	3	5
15	19	24	24	12	9	10	16	4	8	18	4	14	23	3	5	22	14	20	16	8	24	12	17	7
16	20	10	22	12	11	2	16	11	8	18	6	15	21	21	13	11	23	21	16	11	24	17	3	5
17	20	15	24	12	10	2	18	6	9	21	21	19	16	12	17	15	1	21	10	5	19	3	5	12
18	23	2	24	20	9	14	18	7	21	8	19	6	16	11	13	15	22	10	5	10	19	17	5	12
ΣR	312	159	359	274	204	144	209	205	162	313	193	215	119	140	318	338	224	272	205	101	349	243	219	123

<u>Note</u>. Students sorted the problems from the one they would most prefer to do to the one they would least prefer to do.
A score of 24 indicates a student's first choice (most preferred) while a score of 1 indicates the student's last choice.

students sort the problems? To answer this question, the Friedman rank
sums test was employed. Developed by economist Milton Friedman in 1937,
the statistic tests the null hypothesis that there is no treatment ef-
fect (H_0: $\tau_1 = \tau_2 = \cdots = \tau_k$) against the alternate hypothesis that the
treatment effects are not all equal (at least one of the treatment ef-
fects is not zero). While the test is appropriate for ordinal data, an
underlying assumption is that the responses must come from a population
of continuous random variables. To compensate for the discontinutiy of
the data, a forced-choice system was employed where ties were not allowed.

The Friedman rank sums test is a substitute for the classical F
test for detecting treatment effects in a randomized block design or a
repeated measures design for large sample sizes. Its sampling distribu-
tion is approximated by a chi-square distribution with k-1 degrees of
freedom. The formula for the Friedman statistic is

$$S = ((12/nk(k + 1)) \sum_{j=1}^{k} R_j^2) - 3n(k + 1),$$

and the approximate α level test is

Reject H_0 if $S \geq \chi^2_{(k - 1, \alpha)}$

Accept H_0 if $S < \chi^2_{(k - 1, \alpha)}$ (Hollander & Wolfe, 1973).

Using the Friedman statistic, a value of 151.98 was computed, which is
statistically significant at all reasonable levels according to its
chi-square approximation. Consequently, the null hypothesis was re-
jected.

An examination of the data showed a marked tendency of students
to prefer problems 3, 16, and 21, the logic problems, over all others

For problem 3, four students made it their first choice, one student made it his second choice, and five students made it their third choice. Similar results were found with the other two "pure" logic problems, numbers 16 and 21. _Pure_ in this instance is intended to denote that the problems were of the same type with respect to content and structure. (See the comments under question two for a fuller discussion of this topic.)

Question Two

In the second question, the study investigated whether students would sort the problem set according to the mathematical content of each problem. Would they order the problems so that grouping by content could be observed? To verify the mathematical content of each problem, a complete set of the 24 problems was given to three members of the mathematics education faculty and one member of the mathematics faculty at the University of Florida. They were asked to sort the problems into four types: number theoretic problems involving an inductive strategy, number theoretic problems involving factors or other properties of the quantities involved, logic problems, and geometry problems. Each person performed the sorting process individually. When their results were combined, two different groupings were observed. Analysis of these groupings showed some confusion between the content and structure of the problems. Webb (1979) pointed out that these are the two categories of the task variables which refer to the "main essence of mathematical problems" (p. 77). He described content variables as the substance of the task, while structure

variables describe the models that represent the solution process of the task. The confusion was, therefore, one where, in the absence of clear directions, it was unknown whether the problems should be sorted by content or structure. Follow-up discussions clearly defined the set of problems in terms of both content and structure.

The question of interest was then amended so that grouping by both content and structure could be investigated. The problems were sorted in terms of both content and structure. Table 4-2 gives the mean of the ranks assigned to each problem, the rank of each mean among all the means, and each problem's content and structure type. With respect to content, there were 3 inductive problems, 4 factor problems, 9 logic problems, and 8 geometry problems. With respect to structure, there were 14 inductive problems, 7 factor problems, and 3 logic problems. None of the problems had a geometry structure since they all called for some form of generalization. Three of the problems were "pure" inductive problems, that is being inductive in both content and structure. There were also 3 "pure" factor problems and 3 "pure" logic problems. There were 4 logic (C)-factor (S) problems, 2 logic (C)-inductive (S) problems, 8 geometry (C)-inductive (S) problems, and 1 factor (C)-inductive (S) problem. To investigate these groups of problems, a combined mean was developed by summing the rankings of all problems in the group and then dividing by the total number of rankings for the group (Table 4-3).

An examination of the content categories alone reveals that those problems that had a logic content had a combined mean of 16.01, those problems with an inductive content had a combined mean of 14.43, the

Table 4-2

Mean Ranking for All Problems

Problem	Mean ranking	Standard deviation	Problem preference	Content type	Structure type
1	17.33	6.47	6	I	I
2	8.83	4.51	19	G	I
3	19.94	5.15	1	L	L
4	15.22	5.76	7	L	F
5	11.33	4.23	16	G	I
6	8.00	5.05	20	F	F
7	11.61	5.75	13	L	I
8	11.39	5.48	14-15 (tie)	F	F
9	9.00	4.39	18	G	I
10	17.39	5.07	5	L	F
11	10.72	4.91	17	F	I
12	11.94	5.99	12	L	I
13	17.67	6.06	4	L	F
14	6.61	6.12	23	G	I
15	7.79	5.61	21	G	I
16	18.79	5.95	3	L	L
17	12.44	6.80	9	I	I
18	15.11	7.14	8	F	F
19	11.39	5.03	14-15 (tie)	G	I
20	5.61	4.54	24	G	I
21	19.39	5.57	2	L	L
22	13.50	5.31	10	I	I
23	12.17	8.18	11	L	F
24	6.83	5.22	22	G	I

Note. F = number theoretic problems that could be solved using factors or other properties of the quantities involved; G = geometry problems; I = number theoretic problems that could be solved using an inductive strategy; L = logic problems.

Table 4-3

Mean Average of Ranked Data by Category

Category	Number of problems	Problems	Mean	Standard deviation
Content				
Logic	9	3,4,7,10,12,13,16,21,23	16.01	6.66
Induction	3	1,17,22	14.43	6.47
Factor	4	6,8,11,18	11.26	6.19
Geometry	8	2,5,9,14,15,19,20,24		
Structure				
Logic	3	3,16,21	19.37	5.48
Induction	14	1,2,5,7,9,11,12,14,15,17, 19,20,22,24	10.35	6.08
Factor	7	4,6,8,10,13,18,23	13.85	6.87
Combinations				
Pure combinations				
Logic	3	3,16,21	19.37	5.48
Induction	3	1,17,22	14.43	6.47
Factor	3	6,8,18	11.50	6.54
Multiple combinations				
Logic (C) - Factor (S)	4	4,10,13,23	15.61	6.62
Logic (C) - Induction (S)	2	7,12	11.78	5.79
Geometry (C) - Induction (S)	8	2,5,9,14,15,19,20,24	8.42	5.27

Note. C = content; S = structure.

factor content problems had a combined mean of 11.26, and the geometry-content problems had a combined mean of 8.42. With respect to the structure categories, the three logic problems had a combined mean of 19.37, the factor-structure problems had a combined mean of 13.58, and the inductive-structure problems had a combined mean of 10.35.

In the "pure" combination categories, the combined mean for the logic problems was 19.37, with 14.43 for the inductive problems and 11.50 for the factor problems. For the multiple combination categories, logic (C)-factor (S) problems had a combined mean of 15.61, with 11.78 for the logic (C)-inductive (S) problems, and 8.42 for the geometry (C)-inductive (S) problems. The geometry-inductive category is identical (same elements) to the geometry-content category since all the problems has an inductive structure. The single factor (C)-inductive (S) problem had a mean of 14.27.

These data indicate that the students most preferred to do logic problems and least preferred to do geometry problems. Inductive problems were more preferred than factor problems. This lower preference for number theoretic problems that could be solved using factors and other properties of the quantities involved could be viewed in terms of the general complexity of this type of problem. In her research, Kantowski (1980) found that these same students relied most heavily on trial and error whenever these types of problems were encountered. This inability to find the "trick" could well have made these problems less popular than, say, the logic problems, which can usually be solved with a chart or diagram and which lend themselves to an "easier" solution process. Here the term _easier_ is used in the sense that the starting point in the solution is known.

Question Three

The Myers-Briggs Type Indicator (MBTI) consists of four dichoto-
mous scales which describe a person's personality preference. Since
each of these four scales has two opposing functions (extraversion vs.
introversion, etc.), there are eight separate type elements (personality
characteristics). The question of interest, therefore, is, if the
students are grouped by type element will each group sort the problems
in the same way? For example, will those students with a preference
for introversion, as a group, sort the problems in a similar manner?

In addition, certain combinations of type elements which were
considered to be interesting were investigated. For instance, the in-
trovert is noted for his preference for the inner world of concepts
and ideas. If this same person prefers to perceive his environment
with his intuition, then he will prefer an indirect perception by way
of the subconscious, accompanied by the ideas and associations which
the subconscious tacks onto that which is perceived. In other words,
his preference is for a totally internal system for dealing with his
environment. It was speculated that INs would tend to order the prob-
lems in the same way. Extraverted-sensing types were also investigated.

The null hypothesis tested was that there was no pattern to the
ordering process within each group versus the alternate hypothesis
that there was some agreement within each group. The Friedman statis-
tic (S) was computed for each group. Eight of the 10 groups investi-
gated had test statistic values sufficiently large to be statistically
significant at the α = .01 level ($\chi^2_{(23, .01)}$ = 41.64). In these 8
cases the null hypothesis was rejected and the alternate hypothesis

that there was some agreement as to the ordering of the problems for the subjects with a preference for introversion (S = 101.64), extraversion (S = 59.38), sensing (S = 118.91), intuition (S = 44.06), feeling (S = 143.38), judgment (S = 72.16), perception (S = 120.43), and introverted intuitives (S = 71.77) was accepted. The 2 groups where the test statistic failed to exceed the critical chi-square value, those with a preference for thinking and the extraverted sensing types, both had a small number of subjects (n = 3), which may have contributed to the lack of significance (Table 4-4).

Question Four

Once the question of intragroup similarities had been investigated, the next step was to determine if persons with opposite type elements possessed different patterns for sorting the problem set. For example, do persons preferring introversion sort the problems in an order different from that of persons preferring extraversion? If persons with different personality preference do sort the problems in different order, then it may be possible to design curricula for problem solving based on individual preference styles.

To detect any differences that may have occurred, Spearman's rank correlation coefficient (rho) was used. The null hypothesis was that there were no differences (H_o: $\rho = 0$) versus the alternate hypothesis that there were some differences (H_o: $\rho \neq 0$). The test statistic employed was

$$\rho = 1 - (6 \sum_{i=1}^{N} d_i^2/(N(N-1))),$$

where d is the difference between paired ranks and N is the number of cases ranked. The null hypothesis would be rejected if ρ was too

Table 4-4

Friedman Statistic Results for Each of the Eight MBTI Type Elements

Type element	Number of subjects	Friedman statistic
Introversion (I)	11	101.64^a
Extroversion (E)	7	59.38^a
Sensing (S)	11	118.91^a
Intuition (N)	7	44.06^a
Thinking (T)	3	20.46
Feeling (F)	15	143.38^a
Judgment (J)	6	72.16^a
Perception (P)	12	120.43^a
IN	7	71.77^a
ES	3	17.79

[a]Statistically significant at the α = .01 level.

large or too small, that is, reject H_0 if $|\hat{\rho}| > C_{\alpha/2}$ is the critical value at the $\alpha/2$ level. The closer to zero the statistic, the less the amount of agreement there is in the ranked data. The closer the statistic is to 1, the greater the amount of agreement there is in the ranked data. A value of +1 would indicate perfect positive correlation between the two sets of ranked data, while a score of -1 would indicate a perfect negative correlation. The results are reported in Table 4-5.

Table 4-5

Spearman rho Values for Detecting Intergroup Differences
for the Dichotomous MBTI Scales

Scale	rho
Extraversion (E) vs. Introversion (I)	.904
Sensing (S) vs. Intuition (N)	.850
Thinking (T) vs. Feeling (F)	.391
Judging (J) vs. Perceiving (P)	.880
Introverted-intuitives (IN) vs. Extraverted-sensing (ES)	.661

The critical value for α = .01 if $C_{\alpha/2}$ = $C_{.005}$ = .625. Therefore, the null hypothesis of no correlation is rejected for four of the five scales tested. For the four scales (extraversion vs. introversion, sensing vs. intuition, judging vs. perceiving, and introverted-intuitives vs. extraverted-sensing types) there are no grounds for assuming that any differences in order can be attributed to type element differences. There was only one scale (thinking vs. feeling) where the test statistic failed to exceed the critical value. While failure to reject

the null hypothesis does not mean that it should be accepted, this find-
ing may indicate that persons who make judgments according to different
criteria may sort problems in different ways. The thinking type prefers
to make judgments using clear logical criteria while persons who prefer
feeling will make their judgments according to some personal set of
values.

Question Five

The last question investigated in this study was to ascertain
the presence of any relationship between individuals on the required
task of sorting by preference to do the problem. This analysis was
accomplished by comparing the ranked data of each student with every
other student in the group. The null hypothesis for each pairing was
that there would be no similarity in the way the two students ordered
the problems (H_o: $\rho = 0$) versus the alternate hypothesis that some
regularity would exist (H_a: $\rho \neq 0$). The test statistic used was
Spearman's rho. The procedures used were the same as those used in
question four, with an $\alpha = .01$ and $C_{\alpha/2} = .625$. The results of per-
forming the pairwise comparison are reported in Table 4-6. While there
were several instances where $|\hat{\rho}| \geq C_{\alpha/2}$ and the null hypothesis was
rejected, a post-hoc analysis of the data failed to show any regular
pattern that could be attributed to personality type.

Table 4-6

Spearman's Rank Correlation Coefficient for Pairwise Comparison of Students

Student		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
1.	INTJ	—																	
2.	ENTP	.339	—																
3.	ISFP	.338	.150	—															
4.	ENFJ	.204	.408	-.155	—														
5.	ENFP	.525	.630*	.163	.685*	—													
6.	INFP	.270	.407	.308	-.197	.084	—												
7.	ISFP	.259	.464	-.299	.669*	.673*	-.050	—											
8.	INTP	.226	.138	-.340	.663*	.634*	-.158	.853*	—										
9.	ISFP	.353	.537	-.235	.623	.665*	-.119	.758*	.756*	—									
10.	ENFJ	.523	.520	-.104	.528	.826*	.051	.589	.610	.538	—								
11.	ESTJ	-.022	-.239	.106	-.310	-.221	-.165	-.337	-.369	-.378	-.067	—							
12.	ENFP	.536	.356	.289	.204	.540	.312	.425	.223	.454	.361	-.075	—						
13.	ESFP	.465	.472	.137	.319	.593	.251	.437	.487	.426	.695*	-.075	.320	—					
14.	INFP	.217	.471	-.136	.775*	.664*	-.142	.790*	.755*	.837*	.543	-.417	.279	.417	—				
15.	INFP	.571	.559	-.030	.630*	.829*	.175	.666*	.690*	.613	.852*	-.170	.469	.697*	.683*	—			
16.	ISFJ	.482	.619	.083	.461	.628*	.408	.415	.403	.457	.736*	-.244	.451	.701*	.343	.779*	—		
17.	ESFJ	.178	.305	.161	.271	.414	.092	.261	.291	.561	.269	-.421	.508	.156	.553	.398	.519	—	
18.	INFP	.607	.561	.149	.053	.494	.494	.097	-.025	.120	-.683*	.164	.363	.464	.017	.402	.609	.157	—

$*|\beta| > C_{\alpha/2} = .625$ where $\alpha = .01$.

CHAPTER FIVE
SUMMARY AND CONCLUSIONS

The purpose of this study was to investigate possible relationships between students' personality preferences and the type of mathematics problems they preferred to do. The Myers-Briggs Type Indicator (MBTI) was used to assess personality preferences, and a set of 24 non-routine mathematics problems was used to assess perceived discrimination among problem types.

The MBTI consists of four dichotomous indices which when used together aim to present a description of subjects' personality styles. The four scales are

Index	Preference
E-I	Extraversion or Introversion
S-N	Sensing or Intuition
T-F	Thinking or Feeling
J-P	Judgment or Perception

The eight personality characteristics are referred to as type elements. Each index is designed to reflect an individual's habitual choice between opposites. Subjects may prefer introversion over extraversion, sensing over intuition, etc., as a natural style. The four indices, usually represented by a four-letter combination (ESTJ, INFP, etc.), attempt to describe the preferences which reflect subjects' personalities. The theory upon which the MBTI is based requires that psychologically healthy individuals be capable of using each index type

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as it becomes appropriate. This means that subjects who prefer intro-
version will be capable of using extraversion in those social situations
requiring its use. For this study, the subjects were grouped by type
element. This means that subjects with a preference for introversion
were grouped together as were those with a preference for extraversion,
etc. Each student, therefore, was represented four times, once for
each index.

A theoretical framework was developed in Chapter Two which traced
the evolution of task variables in problem solving from the work of
Polya (1957) to the four-category system presented by Goldin and
McClintock (1979) (see Figure 2-1). Two of these task variables, con-
tent and structure, were investigated in this study. Of interest was
whether students with different personality styles preferred to do
problems based on the content or structure of the problem.

The 24 nonroutine mathematics problems used in this study were
developed as part of a National Science Foundation research probject
conducted by Kantowski (1980). The problem set was designed to repre-
sent four categories of problems. These included number theoretic
problems that could be solved using an inductive strategy, number
theoretic problems that could be solved using factors or other proper-
ties of the quantities involved, logic problems, and problems in
geometry.

A post-hoc analysis of the problem set indicated some confusion
concerning the proper category for each problem. This confusion was
due to the fact that the above categories can denote not only the con-
tent of a problem but also its structure. It is possible, therefore,

for a problem to have a different content and structure. For example, problem 20 ("Given fifty lines in a plane, no three concurrent and no three parallel, into how many regions do the lines divide the plane?") has a geometry content but an inductive structure. After examination of the problem set, it became clear that the four categories listed above denoted the content of the problems, while the structure of the problems could be described using three of the four categories. Since all the geometry-content problems required some form of generalization, there were no geometry-structure problems.

The five questions in this study investigated patterns that emerged as students sorted the problem set from the problem they would most like to do to the one they would least like to do. Two trends were expected to emerge from this procedure: first, that students would tend to order the problems in similar ways, and, second, that the ordering would show a grouping by either content or structure type.

Question one of this study investigated whether there was a general pattern to the ordering process of the students. The Friedman rank sums test clearly showed a strong pattern to the ordering. The students, as a group, did tend to order the problems in the same way. A post-hoc analysis showed that the sorting did appear to be according to type of problem, both in terms of content and structure. Students most preferred to do problems 3, 16, and 21, which contained both a logic content and structure. They were the 1st, 3rd, and 2nd most preferred problems, respectively.

To answer question two, the pattern of student responses to the sorting task was investigated to ascertain whether the students tended

to sort the problems by content, structure, or a combination of both. With respect to the mathematical content of each problem, the order of preference was logic problems, inductive problems, factor problems, and geometry problems. When the problem set was partitioned by structure, the order of preference was logic, inductive, and factor problems. When the problem set was partitioned by combinations of categories, logic-content problems were still the most preferred. Logic problems, whether in pure or multiple combinations, were preferred over all other types as demonstrated by their being picked as the 1st through the 5th overall choices while possessing 9 of the 13 highest overall means.

This preference for logic problems can perhaps be explained in terms of the nature of the solution process. Knowing that the first step in the solution to a logic problem is to draw a chart or diagram, and that the solution can be obtained by correctly completing the figure, may have made the logic problems more attractive to the students than types of problems with more esoteric methods of solution. It may also have been that the problems seemed easier to do.

Inductive problems not connected to the geometry-content problems were the 2nd most preferred group, while factor problems not connected with the logic-content problems were the 3rd most preferred. Clearly, the least preferred problems were the goemetry-content problems, having mean ranks which placed them in five of the last six positions. Since all the problems required some form of generalization to be successfully solved, if trial and error were the only tool in the student's repertoire of problem-solving strategies, then problems like number 14 ("How many diagonals are there in a concave polygon of 50 sides?") would seem like an impossible task.

In question three, this study investigated whether students with the same personality characteristics would sort the problem set in the same way. To test for intragroup concordants, the Friedman statistic was calculated. For 8 of the 10 groups, the test statistic was sufficiently large to reject the null hypothesis that there was no treatment difference, i.e., that there would be no pattern to the ordering process within the group. Agreement in the order of sorting was found for students with a preference for introversion, extravertion, sensing, intuition, thinking, judging, perceiving, and introverted-intuition. The two personality characteristics where the test statistic failed to exceed the critical value were thinking and extraverted-sensing.

In question four, intergroup differences between the opposite types of each index as to how the problem set would be sorted were investigated. On four of five indices studied, the statistical test failed to support the hypothesis that there would be differences in the way subjects with opposing personality characteristics sorted the problems. The thinking-feeling index was the one index where personality characteristics did seem to affect the sorting process. This may well have been the result of the fact that thinking types make their judgments in an impersonal, analytic manner while feeling types make decisions based on a very personal set of values. Therefore, the differences in the order of sorting may have been an artifact of the sorting task rather than the problems themselves.

In question five, this study investigated whether, on a pairwise basis, subjects tended to sort the problem set in the same order. The Spearman's rank correlation coefficient (rho) was calculated

between each pair of students, and the results are listed in Table 4-6.
A note of caution is advised. With so many tests being conducted, the
probability of at least one Type I error is very great. Even taking
an $\alpha = .01$, the results still seem to suggest a certain amoung of agree-
ment with reference to the ordering. This result, taken with the re-
sults of the other four questions, would indicate that while the stu-
dents in this study did tend to group the problems according to either
the content or structure of the problem, their personality style, as
measured by the Myers-Briggs Type Indicator, was not a discriminating
factor in determining the outcome of the ordering process.

Limitations of the Study

1. This study was exploratory in nature. For this reason, while
a sample of size 18 was sufficient to utilize the approximation to the
chi-square distribution, it limits the ability to generalize the re-
sults. This study should be replicated with a much larger sample be-
fore any conclusions can be drawn. Also, this study used above-average
students for the sample. The sorting task should be replicated for
average and below-average students.

2. The Myers-Briggs Type Indicator is a self-reporting inventory,
and while it has been well validated, it shares the shortcomings of all
self-reporting instruments, namely, that either consciously or
unconsciously, the subjects may not be representing their true per-
sonalities.

3. For future studies, clearer deliniation as to the types of
problems used needs to be made.

Implications for Future Research

1. To understand more thoroughly the process that students use in sorting the problem set, verbal protocols of the sorting process should be investigated. Recording students as they talk aloud while sorting the problems may give a fuller insight as to why one problem or type of problem is preferred over another.

2. While a purpose of this study was to investigate whether students would sort problems according to the content or structure of the problems, no a priori decision was made concerning which problems or types of problems would be most preferred. If there is indeed a pattern to students' preferences, then by assuming this pattern it would be possible to check for monotonicity using such statistical procedures as Page's L. This test for monotonicity would not only tell whether students discriminated between types of problems but also which problems were more preferred.

3. This study used four content-type problems. A further refinement would be to develop a problem set consisting of just one content type of problem. The ordering of this problem set may indicate other affective characteristics or task variables which are important in the problem solving process.

Suggesions for Instruction

1. The types of problems used in this study are those which typically give students a great deal of trouble. Therefore, a thorough understanding of both the problems and the students is necessary for the implementation of effective teaching strategies. Problems should be analyzed in terms of the task variables presented in this study.

Characteristics of the problems should be investigated and stressed during instruction. Several examples of each type of problem should be examined.

2. The inductive nature of problems should be stressed during instruction. The ability to find patterns and generalize are two of the most important concepts which can be taught in the mathematics classroom. All too often, students can solve individual types of problems but never see how a particular solution process may fit into other types of problems.

3. Teachers should become familiar with the personality characteristics of their students. The results of research cited in Chapter Two of this study indicated that individuals' personality preferences do influence and to a large extent determine what and how they will learn. Instruction should be planned so that appropriate teaching strategies are used which meet the needs of all students.

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PROBLEM 1: A number represented by the pattern below is called a triangular number. What would be the next triangular number? What would be the 50th triangular number? Find the Nth triangular number.

```
  •        •          •              •
         • •        • •            • •
                  • • •          • • •
                               • • • •
```

PROBLEM 2: How many squares with whole number dimensions can be counted on a regular checkerboard?

PROBLEM 3: 1.) Joe, Jack, John, and Tom live one on each floor of a four-story apartment.
2.) Their ages are 10, 9, 8, and 5, not necessarily in that order.
3.) Joe lives directly above the 9-year-old and directly below the 8-year-old.
4.) Jack has to pass the 5-year-old to leave the building from his apartment.
5.) Jack is more than one floor away from Tom, who is more than one year younger than Jack.
Find the ages and on which floor each boy lives.

PROBLEM 4: Replace each letter in the pentagon below by a number from 1 to 10 (using each number but once) so that the totals of the three numbers on each of the five sides of the pentagon will be the same. Call the common total T. What are the possible values for T?

```
        A
      B   C
    D       E
    F       G
    H   M   N
```

PROBLEM 5: Suppose a cube is three inches on each edge. It is painted red on all of its six faces. Then it is sliced into one-inch cubes. How many one-inch cubes will there be? How many have three red faces? two red faces? one red face? Generalize for an N by N by N cube.

PROBLEM 6: What is the least positive integer by which 180 should be multiplied so that (A) the product is a perfect square?
(B) the product is a perfect cube?

PROBLEM 7: Examine the following relationships:

$$1^3 = 1^2 - 0^2$$
$$2^3 = 3^2 - 1^2$$
$$3^3 = 6^2 - 3^2$$
$$4^3 = \ldots \text{ etc.}$$

State an expression for 50^3.

PROBLEM 8: A certain make of ball point pen was priced at 50 cents in the store opposite the high school but found few buyers. When, however, the store had reduced the price, the whole remaining stock was sold for $31.93. What was the reduced price?

PROBLEM 9: Enclosures such as those shown below are constructed of stair-patterned sets of square rooms. If N is the number of squares on a side, and T is the number of walls, then, by actual count, the table shown results.

N	T
1	4
2	10
3	18
4	...

Three hundred and seventeen walls are available for constructing such an enclosure, but are not sufficient to complete it. What is the least number of additional walls needed to complete the stair pattern?

PROBLEM 10: Use the numbers 1, 2, 3, 4, 5, and 6 in the triangular pattern below so that the sum along each side of the triangle is the same. What is the smallest such sum? The largest?

PROBLEM 11: What is the number of ways to "divvy-up" 50 pieces of candy among three friends? (i.e. find the total number of ways of putting 50 objects in three piles with at least one object in each pile).

PROBLEM 12: The positive integers are arranged in groups as follows: (1), (2,3), (4,5,6), (7,8,9,10), ... Note that there is one number in the first group, two numbers in the second, and so on. Find a formula for the sum of the k numbers in the kth group.

PROBLEM 13: "How many children have you, and how old are they?" asked the guest, a mathematics teacher.
"I have three boys," said Mr. Smith. "The product of their ages is 72 and the sum of their ages is the street number." The guest went to look at the entrance, came back, and said, "The answer is indeterminate."
"Yes, that is so," said Mr. Smith, "but I still hope that the oldest boy will someday win the Stanford competition." Tell the ages of the boys, stating your reasons.

PROBLEM 14: How many diagonals are there in a convex polygon of 50 sides?

PROBLEM 15: On the "isometric" geoboard below, an equilateral triangle is indicated in the upper left-hand corner. Use this triangular region as the unit of area. Use the side of the triangle as the unit of length. Determine the area of an equalateral triangle with 50 units on each side.

$$\triangledown \quad \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot$$

PROBLEM 16: Five candidates—Ainsworth, Borrow, Coleridge, Defoe, and Emerson—competed for the president's medal at All Saints. There were also five subjects—English, history, Latin, Greek, and philosophy. Marking was on an "ordinal" basis. The candidate that placed first in a subject secured five marks; the candidate that placed second, four marks; the remaining candidates, three, two, and one marks, respectively. Final placings were determined by the aggregate of marks scored. No two candidates tied in any subject. Ainsworth won the medal easily, with the excellent aggregate of 24 marks. Coleridge showed consistency—he secured the same mark in each of four subjects. Emerson took first place in Greek and third place in philosophy. No two candidates secured the same aggregate of marks, their final placing corresponding, as it happened, to the alphabetical order of their names. How many marks did Barrow secure in Greek?

PROBLEM 17: Predict the sum:

$$1^3 + 2^3 + 3^3 + \ldots + 50^3$$

PROBLEM 18: How old is the captain, how many children has he, and how long is his boat: Given the product 32118 of the three desired numbers (integers). The length of the boat is given in feet (several feet), the captain has both sons and daughters, he has more years than children, but he is not yet one hundred years old.

PROBLEM 19: Suppose in a town with only square blocks, a special grass is planted around certain neighborhoods. One of the square neighborhoods is sketched below. It contains nine square blocks. Can you predict how many blocks will touch grass on two sides, one side, or no sides when the size of the neighborhood is N by N?

PROBLEM 20: Given 50 lines in a plane, no 3 concurrent and no 3 parallel, into how many regions do the lines divide the plane?

PROBLEM 21: "My four granddaughters are all accomplished girls," Canon Chasable said with evident satisfaction. "Each of them plays a different musical instrument, and each speaks a European language as well as, if not better than, a native."
"What does Mary play?" asked someone.
"The cello."
"Who plays the violin?"
"D'you know," said Chasable, "I've temporarily forgotten. But I do know its the girl who speaks French."
The remainder of the facts which I elicited were of somewhat negative character. I learned that the organist is not Valerie, that the girl who speaks German is not Lorna, and that Mary knows no Italian. Anethea doesn't play the violin, nor does the girl who speaks Spanish. Valerie knows no French, Lorna doesn't play the harp, and the organist can't speak Italian.
What are Valerie's accomplishments?

PROBLEM 22: Consider the table:

$$
\begin{array}{rcl}
1 & & 1 \\
2 + 3 + 4 & & 1 + 8 \\
5 + 6 + 7 + 8 + 9 & & 8 + 27 \\
10 + 11 + 12 + 13 + 14 + 15 + 16 & = & 27 + 64 \\
\text{etc.} & &
\end{array}
$$

Find the pattern and generalize.

PROBLEM 23: Among grandfather's papers and bills were found

72 turkeys $-67.9-

The first and last digit of the number that obviously represented the total price of the fowl are replaced here by blanks, for they have faded and are now illegible. What are the two faded digits and what was the price of one turkey?

PROBLEM 24: How many lines may be drawn joining 18 points on a circle?

APPENDIX B
DIRECTIONS FOR PROBLEM PREFERENCE TEST

Order the deck of problem cards. Select the problem you <u>would like</u>
to do first, then second, then third, etc., until you have used all
the cards.

Order of preference:

___, ___, ___, ___, ___, ___, ___, ___,

___, ___, ___, ___, ___, ___, ___, ___,

___, ___, ___, ___, ___, ___, ___, ___.

APPENDIX C
STUDENT INFORMATION

Student	Sex	MBTI type	IQ[a]	Grade in school	Previous math courses
1	M	INTJ	148	11	Algebra II (9) Math Analysis (10)
2	F	INTP	142	11	Algebra II (9) Math Analysis (10)
3	M	ISFP	(b)	12	Algebra II (9) Geometry (10) Math Analysis (11)
4	M	ENFJ	136	12	Algebra II (9) Geometry (10) Math Analysis (11)
5	M	ENFP	141	10	Algebra II (9)
6	M	INFP	(b)	11	Algebra II (9) Geometry (10)
7	M	ISFP	136	10	Algebra II (9)
8	M	INTP	133	10	Algebra II (0)
9	M	ISFP	(b)	12	Algebra I (9) Algebra II (10) Geometry (11)
10	F	ENFJ	136	10	Algebra II (9)
11	M	ESTJ	125	12	Algebra II (9) Trigonometry (10) Math Analysis (11)
12	M	ENFP	125	12	Algebra II (9) Geometry (10) Math Analysis (11)
13	M	ESFP	132	10	Algebra II (9)
14	F	INFP	(b)	12	Algebra II (9) Geometry (10) Math Analysis (11)
15	F	INFP	139	9	Algebra II (9)[c]

Student	Sex	MBTI type	IQ^a	Grade in school	Previous math courses
16	F	ISFJ	130	11	Algebra II (9) Math Analysis (10)
17	F	ESFJ	141	10	Algebra I (9) Algebra II (10)[c]
18	F	INFP	145	10	Algebra II (9)

[a]IQ was determined using the Wechsler Intelligence Scale for Children.
[b]Not tested.
[c]Taking course concurrently with problem-solving course.

BIOGRAPHICAL SKETCH

Born on March 31, 1944, in Cambridge, Massachusetts, Steven Franklin Miller lived with his parents, Max and Thelma Miller, and his sister, Wendy. He attended Palm Beach High School in West Palm Beach, Florida, and upon graduation in 1961, enrolled at the University of Florida, Gainesville, Florida. After graduating with the degree of Bachelor of Science in Business in 1967, he enrolled in the graduate program at the University of Florida.

In 1968, he accepted a teaching position at Bradford Middle School in Starke, Florida, where he taught sixth-, seventh-, and eighth-grade mathematics. In 1970, he was appointed mathematics department chairman. In 1973, he transferred to Bradford High School where he taught algebra, trigonometry, math analysis, and calculus.

He received the degree of Master of Education with a major in secondary education and a minor in mathematics in 1974, and began his doctoral studies in mathematics education under the direction of Dr. Elroy J. Bolduc, Jr. During the 1976-1977 school year he took a leave of absence from his teaching duties to attend the University of Florida on a full-time basis, where he served as a graduate teaching assistant, teaching undergraduate courses and supervising mathematics interns. The following year he returned to his high school teaching duties.

Steven Miller is married to the former Julianne Temples.

I certify that I have read this study and that in my opinion it conforms to acceptable standards of scholarly presentation and is fully adequate, in scope and quality, as a dissertation for the degree of Doctor of Education.
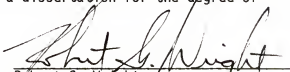
Elroy J. Bolduc, Jr., Chairman
Professor of Subject Specialization
Teacher Education

I certify that I have read this study and that in my opinion it conforms to acceptable standards of scholarly presentation and is fully adequate, in scope and quality, as a dissertation for the degree of Doctor of Education.
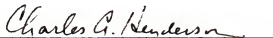
Eleanore L. Kantowski
Associate Professor of Subject
Specialization Teacher Education

I certify that I have read this study and that in my opinion it conforms to acceptable standards of scholarly presentation and is fully adequate, in scope and quality, as a dissertation for the degree of Doctor of Education.

Robert G. Wright
Associate Professor of Subject
Specialization Teacher Education

I certify that I have read this study and that in my opinion it conforms to acceptable standards of scholarly presentation and is fully adequate, in scope and quality, as a dissertation for the degree of Doctor of Education.

Charles A. Henderson
Associate Professor of Subject
Specialization Teacher Education

I certify that I have read this study and that in my opinion it conforms to acceptable standards of scholarly presentation and is fully adequate, in scope and quality, as a dissertation for the degree of Doctor of Education.

Charles W. Nelson
Professor of Mathematics

This dissertation was submitted to the Graduate Faculty of the Division of Curriculum and Instruction in the College of Education and to the Graduate School, and was accepted as partial fulfillment of the requirements for the degree of Doctor of Education.

April, 1983

Dean for Graduate Studies and
Research